Lecture no.13

Basic Concepts about Nonideal Dusty Plasma
(continuation)

Electrostatic potential around a dust particle

The distribution of the electrostatic potential $\varphi(r)$ around an isolated spherical dust particle of charge Z_d in an isotropic plasma satisfies the Poisson equation:

$$\Delta\varphi = -4\pi e(n_i - n_e) \tag{1}$$

with the boundary conditions $\varphi(\infty) = 0$ and $\varphi(a) = \varphi_s$. The potential is connected to the particle charge according to the following relation:

$$\left.\frac{d\varphi}{dr}\right|_{r=a} = -\frac{Z_d e}{a^2} \quad . \tag{2}$$

In the case of plasma with a Boltzmann distribution of electrons and ions, where the condition $\left|e\varphi_s / k_B T_{e(i)}\right| < 1$ is satisfied, we can linearize the right–hand side of Eq. (1) and have the following expression for $\varphi(r)$:

$$\varphi(r) = \varphi_s \cdot \frac{a}{r} \exp\left(-\frac{r-a}{r_D}\right) , \tag{3}$$

where in this case $r_D^{-2} = d_e^{-2} + d_i^{-2}$. The surface potential is connected to the charge through the formula $\varphi_s = \frac{Z_d e}{a}(1 + a/r_D)$. For the case $a \ll r_D$, the expression for the potential distribution can be written in the following form:

$$\varphi(r) = \frac{Z_d e}{r} \exp\left(-\frac{r}{r_D}\right) \quad . \tag{4}$$

It should be noted that the potential (4) is the screened Coulomb potential which is often applied to describe the electrostatic interaction between the particles in dusty plasmas. In different physical systems this form of the potential is also known as the Debye–Hueckel or Yukawa potential.

Main forces acting on dust particles in plasmas

Notice that the main forces acting on dust particles in plasmas can be conveniently divided into two groups:

- the forces which do not depend on the particle charge (force of gravity, neutral drag force, thermophoretic force):
- the forces which depend directly on the particle charge (electrostatic force and the ion drag force).

The gravitational force. The gravitational force is determined by the following expression:

$$F_g = m_d g \ , \tag{5}$$

where g is the gravitational acceleration. The gravitational force is proportional to the particle volume, i.e. $F_g \sim a^3$.

The neutral drag (friction, resistance) force. In the case of weakly ionized plasma, the main contribution to this resistance force comes from the neutral component. The two regimes which are determined by the Knudsen number $Kn = l_n / a$ should be distinguished. Here l_n, a are the atomic or molecular free path and the size of dust particles, respectively. In the case $Kn \ll 1$ (the hydrodynamic regime) the resistance force is determined by the Stokes expression:

$$F_n = -6\pi\eta a u \ , \tag{6}$$

where η is the neutral gas viscosity, and u is the particle velocity relative to the neutral gas. In the opposite limiting case of $Kn \gg 1$ (the free molecular regime) and for sufficiently small relative velocities ($u \ll \upsilon_{T_n}$) we have the following formula:

$$F_n = -\frac{8\sqrt{2\pi}}{3}\delta a^2 n_n T_n \frac{u}{\upsilon_{T_n}} \quad , \tag{7}$$

where n_n and T_n are the concentration and temperature of the neutrals; δ is a coefficient on the order of unity that depends on the exact processes proceeding on the particle surface. In the case of high relative velocities ($u \gg \upsilon_{T_n}$) the neutral drag force is determined as follows:

$$F_n = -\pi a^2 n_n m_n u^2 \quad , \tag{8}$$

where m_n is the mass of neutrals.

The thermophoretic force. If a temperature gradient takes place in a neutral gas, then the particle experiences a force directed opposite to this gradient, i.e., in the direction of lower temperatures. It is connected with fact that the larger momentum is transferred from the neutrals coming from the higher temperature region.

$$F_{th} = -\frac{4\sqrt{2\pi}}{15}\frac{a^2}{\upsilon_{T_n}}\kappa_n \nabla T_n \quad , \tag{9}$$

where κ_n is the thermal conductivity coefficient of the gas.

The electrostatic force. The electrostatic force acting on conducting charged particles is given by the following formula:

$$\vec{F}_{el} = Z_d e \vec{E}_{eff} \quad , \tag{10}$$

where an effective electric field can be expressed as:

$$\vec{E}_{eff} = \vec{E}\left[1 + \frac{(a/r_D)^2}{3(1+a/r_D)}\right] . \tag{11}$$

Plasma polarization induces a dipole moment of dust particles $\vec{p} \approx a^3 \vec{E}_{eff}$ directed along the field. In the nonuniform electric field the force acting on dipole has the following form:

$$\vec{F}_{dp} = (\vec{p}\nabla)\vec{E} \quad . \tag{12}$$

Ion drag force. If we have a drift of ions (electrons) relative to the dust particle, then there is a force connected with the momentum transfer from the plasma to the dust particle. Due to the larger ion mass, the effect associated with the ions typically dominates, therefore, this force is called “ion drag force”. The ion drag force is connected with two processes: momentum transfer from the ions that are collected by the particle (non-elastic scattering), and momentum transfer from the ions that are elastically scattered in the electric field of the particle.

In the general case the formula for the ion drag force is written as:

$$\vec{F}_I = m_i n_i \int \vec{v} f_i(\vec{v}) \sigma_i^{tr}(\upsilon) \upsilon d\vec{v} \quad , \tag{13}$$

where $f_i(\vec{v})$ is the ion velocity distribution function, and $\sigma_i^{tr}(\upsilon)$ is the momentum transfer cross–section for ion collisions with the dust particle.

It should be noted that at the present time most of the results have been obtained for binary collision (BC) approximations, i.e., for the case **of collisionless ions** with and “isolated” dust particles.